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Prediction of Load and Power Fluctuations from Wind Turbine Spinner-integrated Wind LIDAR Measurements:

by

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A simple model for the fluctuating loads on the blade tip:

The lift force F_{Lift} on a section of a wind turbine's blade is given by the lift equation

$$F_{Lift} = c_L ½ \rho U^2{}_{Lift} \tag{1.1}$$

where c_L is the lift coefficient, and where $½ \rho U^2{}_{Lift}$ is the dynamic pressure force at the stagnation point on the blade front.

For operation far below the stall region the lift coefficient is to first order proportional to the angle of attack[1], α, hence we might write $c_L = \tilde{c}_L \alpha$, where we have factored the angle of attack dependence out from in the lift coefficient.

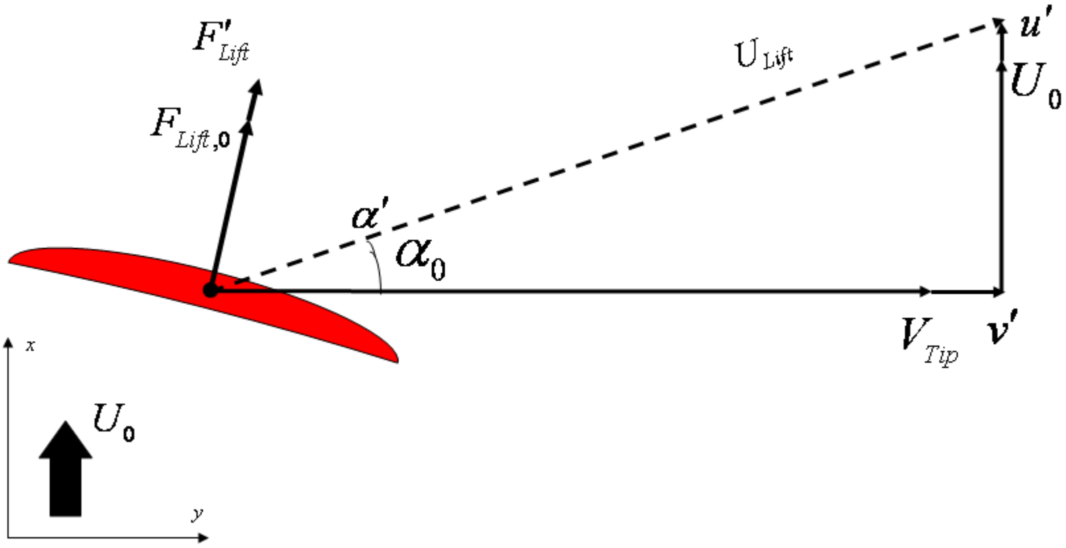

Figure 1. A simple model for t*he lift force on a blade tip. The tip (pointing vertical and top viewed) is moving to the right with the tip speed V_{Tip} in a turbulent flow with mean wind speed U_0 in the x direction. We consider two turbulence components, one in the mean wind direction u' and one in the (tangential) tip ward direction v'.*

[1] In this simplified model where it is assumed that the "angle of attack" is linearly related to, and controllable by the pitch angle of the blade.

We may decompose the lift force into a mean and a fluctuating part, viz.:

$$F_{Lift} = F_0 + F'$$

and expand:

$$\begin{aligned} F'_{Lift} &= \frac{dF_{Lift}}{d\alpha}\alpha' + \frac{dF_{Lift}}{dU_{Lift}}U'_{Lift} \\ &= F_{Lift}\frac{\alpha'}{\alpha} + 2F_{Lift}\frac{U'_{Lift}}{U_{Lift}}; \\ \frac{F'_{Lift}}{F_{Lift}} &= \frac{\alpha'}{\alpha} + 2\frac{U'_{Lift}}{U_{Lift}} \end{aligned} \tag{1.2}$$

That is, there are two contributions to the load fluctuations, one from the changes in angle of attack, α', another from the fluctuations in the lift speed U'_{ift}

1. Fluctuations in "angle of attack":

From Fig. 1 we immediately have:

$$\alpha = \alpha_0 + \alpha' = \tan^{-1}\left(\frac{U_0 + u'}{V_{Tip} + v'}\right) \approx \frac{U_0}{V_{Tip}} + \frac{u'}{V_{Tip}} - \frac{U_0 v'}{V_{Tip}^2};$$

Hence

$$\alpha_0 \approx \frac{U_0}{V_{Tip}}, \tag{1.3}$$

and

$$\alpha' \approx \frac{u'}{V_{Tip}}$$

2. Fluctuations in the lift speed:

Next, by use of the relations $U_o >> u';\ V_{Tip} >> v',\ V_{Tip} >> U_0$ we can expand terms to first order:

$$
\begin{aligned}
U_{Lif\,t}^2 &= \left(U_0 + u'\right)^2 + \left(V_{Tip} + v'\right)^2 \\
&= U_0^2 + V_{Tip}^2 + 2(U_0 u' + V_{Tip} v') + u'^2 + v'^2 \\
&\approx V_{Tip}^2 + 2V_{Tip} v';
\end{aligned}
$$

But since, also to first order:

$$U_{Lift}^2 = U_{Lift,0}^2 + 2U_{Lift,0} U'_{Lift}$$

we get:

$$U_{Lift,0} \approx V_{Tip}$$

and from

$$2U_{Lift,0} U'_{Lift} \approx 2V_{Tip} v'$$

we get

$$2\alpha_0 \qquad U'_{Lift} = \frac{V_{Tip}}{U_{Lift,0}} v' \approx v' \tag{1.4}$$

Combined fluctuation model

Combining the above two expressions into(1.2), we find that the relative fluctuations in lift force can be written as:

$$
\begin{aligned}
\frac{F'_{Lift}}{F_{Lift}} &= \frac{\alpha'}{\alpha} + 2\frac{U'_{Lift}}{U_{Lift}} \\
&\approx \frac{u'}{V_{Tip}\alpha} + 2\frac{v'}{U_{Lift}} \\
&\approx \frac{u'}{U_0} + 2\frac{v'}{V_{Tip}} \\
&\approx \frac{u' + 2\alpha_0 v'}{U_0}
\end{aligned}
\tag{1.5}
$$

During normal wind turbine operations, the mean angle of attack, α_0 will is small, O(0.1), and the relative fluctuations in the lift force will be dominated by the relative fluctuations in the stream wise mean wind speed:

$$\frac{F'_{Lift,u'}}{F_{Lift}} \approx \frac{u'}{U_0}$$

However, the contribution from the tangential wind fluctuations, $\frac{2\alpha_0 v'}{U_0}$, when expressed in terms of tangential turbulence intensities, i_v, to

$$\frac{F'_{Lift,v'}}{F_{Lift}} \approx \frac{2\alpha_0 v'}{U_0} \approx 2\alpha_0\, i_v$$

Measurement of the fluctuating lift force from a spinner-based wind lidar :

The combined fluctuating lift force term, however, $\frac{u' + 2\alpha_0 v'}{U_0}$, can in principle be measured from a single scanning wind lidar, coaxially mounted in the wind turbine spinner, from which position it scans the upwind approaching wind field at high spatial and temporal resolution over the rotor plane.

However, a spinner-based wind lidar measures the radial wind components along the wind lidar beams pointing direction only.

In contrast, for the fluctuating load, the wind component to measure is the wind component that is pointed a small angle δ off the turbine rotor axis and into the tangential direction of rotation of the wind turbine given by

$$\delta = \tan^{-1}(2\alpha_0) \approx 2\alpha_0 \qquad (1.6)$$

This is because a lidar beam pointed in this direction will, when "vector-dotted" with the instantaneous upwind wind vector, measure the fluctuating lift force term directly

$$\frac{F'_{Lift}}{F_{Lift}} \approx \frac{u' + 2\alpha_0 v'}{U_0} \qquad (1.7)$$

By assuming continuity in the flow field the term $\frac{u' + 2\alpha_0 v'}{U_0}$ is obtainable from a fast spinner or hub- integrated CW wind lidar that at scan rates less than ~1 second scans the entire 2-D radial wind components in the rotor plane at a fixed upwind distance.
The upwind distance can be between 0.5 and 2 rotor diameters upwind, and from these 2-D scans of radial wind measurements calculates the axial and radial wind components in the measured wind field, assuming continuity in the flow field. That is, from upwind scanned rotor plane radial wind measurements, obtainable e.g. with the Risø DTU newly designed and patented[2] 2-D scan head, the fluctuating wind components responsible for the lift force fluctuations u' and v' can be calculated, e.g. by use of a mass-consistent flow model applied to the flow in the rotor plane.

This spinner-based scanning wind lidar approach, however, relies in addition to the continuity assumption also on the applicability of Taylor's frozen turbulence hypothesis to apply for advection of the flow field from the upwind measurement plane to the rotor plane. This conjecture is now being tested experimentally at Risø DTU.

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[2] PCT WO 2009/155924 A1

Predicting the Power Fluctuations

If the wind turbine rotates at constant rpm the wind turbine power production from the tip segment is $\omega\tau$, where

ω is the (assumed constant) angular velocity and τ is the torque. If the tip segment in Fig.1 is located at radial distance r from the spinner then the power production from that segment, located at (r,θ) is

$$P(r,\theta) = \omega r F_{Lift} \cdot Cos(\beta) \quad [Watt] \tag{1.8}$$

where β is the angle between the lift vector $\mathbf{F_{Lift}}$ and the rotor plane.

The total instantaneous power of the wind turbine is then attainable by integrating over the rotor plane, that is, in a polar coordinate system, where radius is r and azimuth angle is θ, we have:

$$P_{WT} = \int_0^{2\pi}\int_0^R P(r,\theta)\, rdrd\theta = Cos(\beta)\omega \int_0^{2\pi}\int_0^R r^2 F_{Lift}(r,\theta)\cdot drd\theta \;\; [Watt]. \tag{1.9}$$

We can calculate the fluctuating part of this quantity by subtracting the means and obtain an expression for the power fluctuations, as:

$$P'_{WT} = \int_0^{2\pi}\int_0^R P'(r,\theta)\, rdrd\theta = Cos(\beta)\omega \int_0^{2\pi}\int_0^R r^2 F'_{Lift}(r,\theta)\, rdrd\theta \tag{1.10}$$

By inserting the simple model for the wind load fluctuations, Eqs. (1.7), we obtain:

$$P'_{WT} = \frac{Cos(\beta)\omega}{U_0}\int_0^{2\pi}\int_0^R r^2 \left(u'(r,\theta) + 2\alpha_0 v'(r,\theta)\right) F_{Lift}(r,\theta)\cdot drd\theta \tag{1.11}$$

We can now also find an expression for the variance of the fluctuating wind turbine power, viz.:

$$\left\langle P'^2_{WT} \right\rangle = \frac{Cos^2(\beta)\omega^2}{U_o^2} \left\langle \int_0^{2\pi}\int_0^R r'^2 \left(u'(r',\theta') + 2\alpha_0 v'(r',\theta')\right) F_{Lift}(r,\theta')\cdot dr'd\theta' \times \int_0^{2\pi}\int_0^R r''^2 \left(u'(r'',\theta'') + 2\alpha_0 v'(r'',\theta''')\right) F_{Lift}(r'',\theta'')\cdot dr''d\theta'' \right\rangle \tag{1.12}$$

an expression (for numerical evaluation) that involves the co-variances in the wind fluctuations of the type $\langle u'(r',\theta')v(r'',\theta'')\rangle$ etc.

Predicting the Lift Force Fluctuations

Similarly, for the lift forces, we have from Eqs. (1.7):

$$F'_{Lift}\left(\theta,r\right)=U_0^{-1}F_{Lift}(r,\theta)\left(u'\left(r,\theta\right)+2\alpha_0 v'\left(r,\theta\right)\right) \tag{1.13}$$

Correspondingly, we have for the lift force covariance function in the 2-dimensional rotor plane:

$$\left\langle F'^2_{Lift}\right\rangle = \frac{F^2_{Lift}}{U_0^2}\left\langle\left(u'\left(r',\theta'\right)+2\alpha_0 v'\left(r',\theta''\right)\right)\times\left(u'\left(r'',\theta''\right)+2\alpha_0 v'\left(r'',\theta''\right)\right)\right\rangle \tag{1.14}$$

The load fluctuations are also seen to be depending on covariance functions of the wind fluctuations $\left\langle u'(r',\theta')u'(r'',\theta'')\right\rangle, \left\langle v'(r',\theta')v'(r'',\theta'')\right\rangle$ and $\left\langle u'(r',\theta')v'(r'',\theta'')\right\rangle$, which all are quantities that can be simulated, e.g. by the HAWC2 simulation tool at Risø DTU, and the quantities can also be measured with the spinner integrated upwind looking 2-D wind scanning lidar.

In homogeneous and stationary turbulence, these wind covariance functions depend only upon the differences in the independent variables, but here, in the surface layer, with strong shear etc, this simplification cannot be expected to apply in general. We must therefore resort on numerical evaluation of these expressions.